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A Curriculum Project on Expressions and Equations in Mathematics 8 Aligned to the Common Core State Standards

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**A Curriculum Project on Expressions and Equations in Mathematics 8 Aligned to the
Common Core State Standards**

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A thesis submitted to the Department of Education and Human Development of

The College at Brockport, State University of New York,

in partial fulfillment of the requirements for the degree of

Master of Science in Education

May 27, 2016

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Abstract

In response to the implementation of the Common Core State Standards (CCSS), this curriculum project was designed to help teachers with the transition to these standards. This curriculum project provides a unit plan on solving equations for the eighth grade curriculum, which is aligned to the Common Core State Standards. The unit plan addresses expressions and equations in alignment with, Mathematical Practice Standard 8. The curriculum project may act as a resource to assist teachers in the process of converting their unit plans to align with the Common Core State Standards.

Chapter 1

Introduction

The concern for mathematics education dates back to the 1980s; numerous documents expressed the concern that students in the United States were performing poorly. As the world became more technological, poor mathematics performance was alarming – a problem that needed to be addressed (Ellis and Berry, 2005). Many people recognized this and decided the United States (US) needed a standard reform and to have consistent learning goals across states. In 2009, the state school chiefs and governors that comprise the Council of Chief State School Officers, also known as CCSSO and the National Governors Association center, or NGA center, took on the assignment to begin a state-led effort to develop the Common Core State Standards, also called the CCSS (CCSS webpage). “The Common Core standards released in 2010 represent an unprecedented shift away from disparate content guidelines across individual states in the areas of English language arts and mathematics” (Porter, et al., 2011). The current president of the US, Barack Obama, has continued a reform in education, and has stated his strong commitment to academic standards as part of his educational reform agenda (Mathis, 2010). In the article, “The “Common Core” Standards Initiative: An Effective Reform Tool”, Mathis (2010) shares the White House Statement given by President Barack Obama on February 22, 2010:

Because economic progress and educational achievement go hand in hand, educating every American student to graduate prepared for college and success in a new work force is a national imperative. Meeting this challenge requires that state standards reflect a level of teaching and learning needed for students to graduate ready for success in college and careers. (p. 1).

The national government has held states accountable for making this educational change through adopting college and career ready standards in reading and mathematics (Mathis, 2010). Mathis (2010) stated: “The Obama administration called for federal Title I aid to be withheld from states that do not adopt these or comparable standards” (p. 26).

A curriculum project on Equations in the Mathematics 8 curriculum is appropriate during this time of educational reform. Algebra can be known as the “gatekeeper” of mathematics. Algebra is a critical tool in order to be able to further study mathematics. The Equations study in the Mathematics 8 curriculum furthers the students understanding of equations and expressions from previous years and also leads students to high school algebra, as well as furthering their education and also in future employment opportunities. On page 11, McCallum (2012) illustrates this path through math, which prepare students for college and career readiness through the flow chart.

In the article, “Middle School Mathematics Teachers’ Knowledge of Students’ Understanding of Core Algebraic Concepts: Equal sign and Variable”, by Asquith, Stephens, Knuth, Alibali (2007) it is stated that:

By viewing algebra as a strand in the curriculum from pre-kindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more sophisticated work in algebra in the middle grades and high school (Nation Council of Teachers of Mathematics [NCTM], 2000, p. 37).

Flows Leading to Algebra

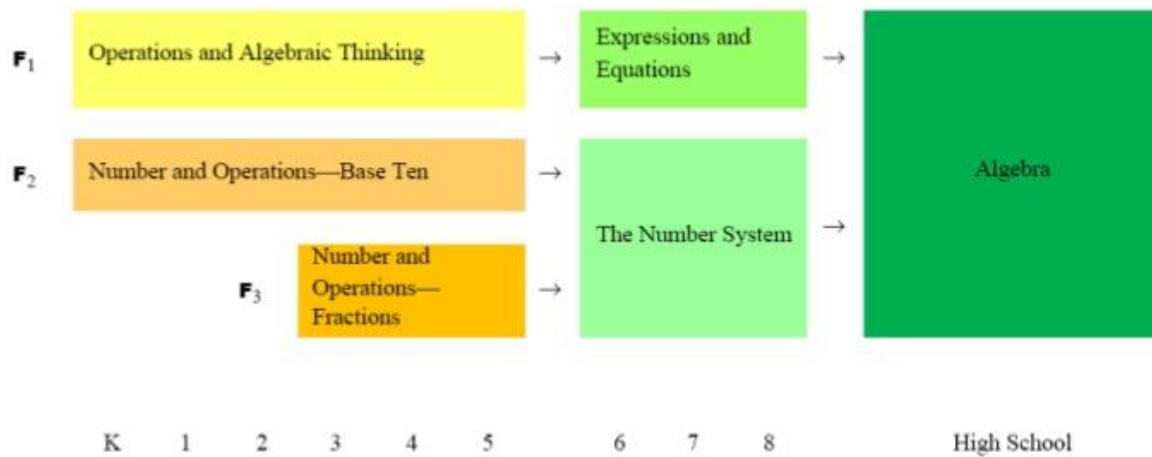


Figure 1: Flows Leading to Algebra- McCallum, W. (2012). (The U.S. Common Core State Standards in Mathematics, Page 11).

Chapter 2

Survey of Literature

Standards Based Education

Educational standards are the learning goals of what students are expected to know and be able to do throughout their education from grades K-12. These standards guide educators in creating appropriate curriculum and to ensure their students are gaining the right skills and knowledge to be successful. Standards can also be a resource for parents to see what will be expected of their children in each grade level. Every state has had educational standards for many years. A 2010 study conducted by the American Institutes of Research, documented a huge expectations gap. Some states were expecting their students to accomplish far more in school than students in other states with much lower standards (Conley, 2014). In other words, a sixth grade student in one state could be expected to understand a dramatically different curriculum than a sixth grade student of another state. The CCSS are an effort to increase student readiness for college and/or career in the economy in this day and age. As many of us have seen, society and the economy are changing very quickly and constantly. The CCSS “help educators create consistency of expectations, clarity of learning targets, and economies of scale in the production of instructional materials carefully crafted to support student success” (Conley, 2014).

The CCSS were created to reform the standards that were in place. NCTM standards were replaced by the CCSS. NCTM was an organization that was founded in 1920 but the standards of NCTM were during the 1980s and 1990s (McLeod, 2003). In the early years of NCTM, the organization was mainly focused on supporting mathematics teachers through the exchange and promotion of good ideas, not through its influence on educational policy (McLeod, 2003). It was not until late 1970s that the organization became an activist rather than a passive

organization. The standards and materials were all about drilling the concept or computation and then practicing it many times. Although the NCTM standards have been reformed or “replaced” by the CCSS, in all of my research, Shirley Hill, president of NCTM from 1970-1980, had one of the best quotes in her *Agenda* for changing curriculum and reforming education.

In the 1960s we learned that curriculum change is not a simple matter of devising, trying out, and proposing new programs. In the 1970s we learned that many pressures from both inside and particularly outside the institution of the school, determine goals and directions and programs....A major obligation of a professional organization, such as ours is to present our best knowledgeable advice on what the goals and objectives of mathematics education ought to be...In my opinion, we are approaching a crisis stage in school mathematics. Policy makers in education are not confronting the deepest problems because the public and its representatives have been diverted by a fixation on test scores...We are still battling against an excessive, narrowing of the curriculum in the name of “back to the basics” (McLeod, 2003).

Shirley Hill’s *Agenda* may not have been a complete guide but this quotation from it, I still believe to hold true. As we see firsthand from the shift to the CCSS, changing curriculum is not a simple process. There are also similarities as to why the different standards have been created. The NCTM standards project was a result of the increased concern about the quality of education. The CCSS project was first started partially because there was increased concern of the education US students were receiving, compared to students from other countries.

As comes with most change, there were some controversies around the CCSS and also when the NCTM standards were coming about. Extra support is usually necessary and there are many discussions of pros and cons around the standards. Throughout the history of mathematics,

curriculum leaders felt that “mathematics was too abstract and too divorced from the real world of work to be of any use to the vast majority of high school students” (Angus, et al., 2003). Many people obviously felt very differently and disagreed with the statement, hence mathematics is still in the high school curriculum. The position of mathematics in the curriculum was at its peak around 1910 and it declined almost continuously until the mid-1970s” (Angus, et al., 2003). The time after 1970 was when Shirley Hill became president of the NCTM organization. During the time of mathematics declining, the nature of mathematics offerings was also changing. Rather than finding more effective ways to introduce students to the mysteries of algebra and geometry, math educators developed high-interest, low-ability alternatives. Standards based education has led the way to abandon the

approach of shunting the “less talented” students off onto tracks leading to nowhere and devote all our energies to developing those multiple instructional strategies that will be necessary to give all American young people realistic access to the kinds of knowledge that both effective citizenship and worthwhile work will require in the twenty-first century (Angus, et al., 2003).

This leads education to the creation of standards and furthermore, the creation of the CCSS, to keep up with the societal changes and the rapid economic changes we go through. As the times change so does education. Below is a chart used to show the changes in education before standards, during the movement, and now under the common core (Kendall, 2011). Below is a table given to easily compare the differences in education during the different times and different standards.

Standards Comparison Chart

	Before Standards-Based Education	During the Standards Movement	Under the Common Core
<i>Appropriateness of expectations to instructional time available</i>	Time available = time needed.	Varies by state; no explicit design criteria. Often, not enough instructional time available to address all standards.	Standards are designed to require 85 percent of instructional time available.
<i>Curriculum support</i>	Curriculum is defined by the textbook.	Standards drive the curriculum, but curriculum development lags behind standards development.	Standards publication is followed quickly by curriculum development.
<i>Methods of describing student outcomes</i>	Seat time; Carnegie units (emphasis on inputs over outcomes).	State standards; criterion-based.	Cross-state standards; consortia of states.
<i>Source of expectations for students</i>	The expectations in textbooks or those described in Carnegie units; historical, traditional influences.	Varies by state; over time, moved from traditional course descriptions to college- and career-ready criteria.	The knowledge and skills required to be college- and career-ready; international benchmarks; state standards.
<i>Primary assessment purposes</i>	Infrequent comparison of students against a national sample; minimum competency tests in the 1970s.	Accountability; to clarify student performance by subgroup (NCLB).	Accountability; to inform and improve teaching and learning.
<i>Systemic nature of reform</i>	Not systemic; reform is enacted through programs at the school or district level.	Reform varies by state and within states. Some are tightly aligned; "local control" states are much less systemic.	Standards, curriculum, and assessment are shared among participating states and territories.

Table 1: Standards Comparison Chart-Kendall (2011). (A Comparison of Education before Standards-Based Education, During the Standards Movement, and Under the Common Core, page 4).

Paradigm Shift from NCTM to CCSS

Forty-three states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) have voluntarily adopted and are moving forward with the standards (CCSS website). The CCSS have tried to fix the problem of having many states covering different topics at different levels in the students' education. The CCSS is mathematically coherent and help lead to career and college readiness at an internationally competitive level. The CCSSO and NGA recommended states "upgrade state standards by adopting a common core of internationally benchmarked standards in math and language arts for grades K-12 to ensure that students are equipped with the necessary knowledge and skills to be globally competitive" (Conley, 2014). The Common Core standards represent the creation of a national curriculum in math and ELA. These standards are designed to create similarity of content across states. One of the differences between state standards and the Common Core standards is the emphasis on instructional technologies, such as calculator use. I found this to be incredibly interesting because Common Core has few to no standards regarding calculators, whereas most school districts allow calculator use and encourage it, even on simple algebra problems. The TI-Nspire has become quite the commodity and are used everywhere. When the students are taught to use these, they are taught how to use the calculator, not the mathematical procedures or concepts. Understanding the structure of the CCSS is key to making this paradigm shift from the NCTM standards to the CCSS effective.

The Structure of the CCSS

A deep understanding of the differences in the CCSS from previous standards is essential in order to properly implement the standards. In order to shift our curriculum from "a mile wide, an inch deep" the key shifts that the CCSS present are crucial. These key shifts in mathematics

are greater focus on fewer topics, coherence, and rigor (www.coresandards.org, 2016, pg. 1). These key shifts are called for by the CCSS and when implemented correctly are supposed to better prepare our students and also to be more competitive internationally. The CCSS push teachers to stop rushing through the material and to cover many topics rather than taking the time and energy to deepen the students' understanding of fewer topics. This will help students gain a stronger base of background knowledge, which will help the students inside and out of the classroom. Having this stronger background of knowledge helps throughout the years of their education and through the various levels because, as the CCSS states, mathematics is made up of interconnected concepts. The standards help reinforce concepts through different grade levels and also build new ideas onto the foundations from previous years. The last key shift in mathematics presented by the CCSS is rigor. Rigor refers to the deep, authentic command of mathematical concepts. The goal of CCSS is not to make math harder or to begin introducing topics at earlier grades, it is for students to gain conceptual understanding, procedural skills and fluency, and application. The shifts from old standards to the CCSS were to create standards that would better help students succeed in life after high school, such as in college or starting a career. Due to what the main goal was, the creators decided what students would need to know by graduation to be more ready and worked their way down the grade levels to kindergarten, rather than working their way up. "The Common Core standards represent a modest shift toward higher levels of cognitive demand than are currently represented in state standards" (Porter, et al., 2011). In my research of the CCSS I found two types of standards and how they are connected. There are the Standards for Mathematical Practice and Standards for Mathematical Content. "The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject

matter as they grow in mathematical maturity and expertise” (CCSS website) throughout their education. The Standards for Mathematical Content are described as “a balanced combination of procedure and understanding” (CCSS website). The content standards, which set goals for content to be taught, or the intended curriculum, may be the “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These standards and ideas of practice lead to the next topic of how the materials and concepts to meet the expectations of these standards are presented. There are many teachers and each one different and unique which presents multiple teaching styles and skills. The way the teacher presents the material may change what a student understands. The teachers’ philosophies and perspectives have a factor in the presentation, which effects how the standards are being met.

Perspectives

The way the teacher presents the material can have a huge effect on what the students takes away from the lesson. The way the teachers learn or have had previous experiences with, relate to how they teach the subject. When learning how to teach mathematics we look at current teaching methods and how teachers need to continually learn and develop new strategies, in order to keep up with the ongoing education reform that constantly takes place. The situative perspective provides new ideas on how to adapt to the cultural and environmental changes we continually go through, in order to be effective as an educator. The situative perspective is “how a person learns a particular set of knowledge and skills, and the situation in which a person learns, are a fundamental part of what is learned” (Peressini et al., 2004).

As teachers, you go through many different experiences, all of which create your professional identity, which can always be changing. The situative perspective, “requires teachers to be active participants in the process of construction of knowledge about learners and

designing methods of instruction most conducive to facilitating student's conceptualization of mathematical ideas"(Frykholm, 1998). "A teacher must be skillful at posing questions that challenge student thinking, rephrasing students' explanations in more mathematically sophisticated terms, carefully listening to ideas, knowing when to provide more information, and orchestrating class discussions to ensure all students are participating"(Peressini et al.,2004). These qualities and skills assist the students in performing to the best of their abilities and can also lead to many, productive class discussions. Discussions can be very useful for a teacher and also assist in getting students involved. Teachers can use discussion as a type of informal assessment, to get a sense of where students are and what they are thinking about the topic. Questions can help guide students thinking, which leads to curiosity. Curiosity then leads to discovery, which give us solutions or new methods to various activities or problems. Keeping students interested or tying the mathematical concept to a life interest of theirs can help spark these questions and curiosities. As teachers, it is good to get to know each student to be able to tie in these various interests to spark the students' curiosity and questioning, which leads to having the students more engaged. This theory presents the transfer of learning to be successful instruction, based on teachers' experiences, education, identity, and professional growth. There are ongoing reforms in education which cause teachers to continually develop new strategies and rethink of how they will present the material to their students and the best way to allow students to be successful to the highest of their abilities. "Clearly the reform movement has placed new demands on math teachers and teacher educators" (Frykholm, 1998). Teaching is a profession of ongoing learning. There are constantly new practices to try and many ways to greater develop professionally.

Peressini's article (2004) discusses the situative perspective in three domains. These three domains are mathematics content, mathematics specific pedagogy, and profession identity. .

“The questions, problems, exercises, constructions, applications, projects and investigations in which students engage constitute the intellectual contexts for their mathematical development” (Peressini et al., 2004). . Teachers need to be able to recognize the “preconceptions and background knowledge that students typically bring to each subject” (Hill et al., 2008).

Using these perspectives, research, and interpretations of these standards a unit plan, which was originally created following the NCTM standards, has been revised to match the CCSS.

Chapter 3

Curriculum Design

This Curriculum Project was created for students in eighth grade mathematics. The unit is intended for instruction during the first quarter of the school year. A Unit Overview is provided as well as a Unit Timeline for teachers to be able to plan ahead and manage their time. This suggested 8-day timeline is based on a class period of 75 minutes, and requires students to complete homework following each lesson in order to practice the new material. If the class period is shorter, the teacher can adjust the lesson to fit the needs of their students, taking the amount of class time into consideration. This curriculum design was created to align to the New York State Common Core Standards. The answer keys are provided in Appendix A. In addition to the necessary worksheets and activities, each lesson plan includes the goal for the lesson, the instructional outcomes, and the New York State Common Core learning standards that are addressed.

Unit Overview

Subject Area

Eighth Grade Mathematics

Approximate Time Needed

8 days-Based on 75 minute class periods

New York State Standards Addressed

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Unit Timeline

Day 1	Day 2	Day 3	Day 4	Day 5
Writing, Solving, and Modeling One Step Equations	Solving One and Two Step Equations	Combining Like Terms and The Distributive Property	Solving More Complex Equations	Equation Station
Day 6	Day 7	Day 8		
Identifying Solutions	Review Day	Assessment: Solving Equations		

DAY	Day 1: Writing, Solving, and Modeling One Step Equations
GOAL	Write, solve, and model one step equations
OUTCOME	1.) Given words, students will translate into a mathematical equation 2.) Given an algebraic equation, students will solve based on provided information 3.) Students will model one step equations
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 2: Solving One and Two Step Equations
GOAL	Solve one and two step equations
OUTCOME	1.) Students will solve one and two step equations using the inverse operations 2.) Given an algebraic equation containing addition or subtraction, and multiplication or division, students will be able to solve for the variable
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 3: Combining Like Terms and The Distributive Property
GOAL	Combine like terms and the Distributive Property
OUTCOME	1.) Students will use algebra tiles to model the distributive property. 2.) Students will solve equations by first using the distributive property.
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 4: Solving More Complex Equations
GOAL	Solve equations with variables on both sides

OUTCOME	1.) Students will solve an equation with a variable on both sides of the equal sign.
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 5: Equation Station
GOAL	Practice with solving complex equations
OUTCOME	1.) Students will practice solving complex equations with groups and share their results with the class.
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 6: Identifying Solutions
GOAL	Identify what solutions an equation will have
OUTCOME	1.) Students will be able to determine the number of solutions an equation will have.
NYS LEARNING STANDARD	8.EE.C.7

DAY	Day 7: Review Day
GOAL	Review of solving equations
OUTCOME	1.) Students will review solving equations.
NYS LEARNING STANDARD	Through this review activity, all the standards from this unit will be addressed.

DAY	Day 8: Assessment: Solving Equations
GOAL	Show me what you have learned about solving equations
OUTCOME	1.) Students will complete the Unit Assessment
NYS LEARNING STANDARD	This assessment is aligned to all the unit standards.

Day 1

Writing, Solving, and Modeling Equations

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Objectives/Goals:

Goal: Write, solve, and model one step equations

Instructional Outcomes:

- 1.) Given words, students will translate into a mathematical equation
- 2.) Given an algebraic equation, students will solve based on provided information
- 3.) Students will model one step equations

Materials:

- Smart Board
- Worksheets (see attached)
- Algebra Tiles

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

Procedure:

- 1.) Warm-up: Students will work on completing the warm-up worksheet as they enter the room. Once they have had about five minutes to solve the problems, we will quickly discuss the answers.
- 2.) Notes: As a class, we will discuss the vocabulary words and also complete the notes.

- 3.) Partner Work: Students will work with a partner. The students will use algebra tiles (see below, included in unit plan on page 22) and work through the hands-on lad to model equations.

Name: _____

Solving Equations-Day 1

Using your background knowledge, come up with a definition for each word. Think on your own, pair with a partner and discuss, then we will share.

Expression-

Equation-

Variable-

What are some words used to tell us that we need to assign a variable to an equation or expression?

In your own words, discuss what the difference is between an equation and an expression.

For each operation, create a list of words that tell you when you should use the specific operation.

ADDITION-

SUBTRACTION-

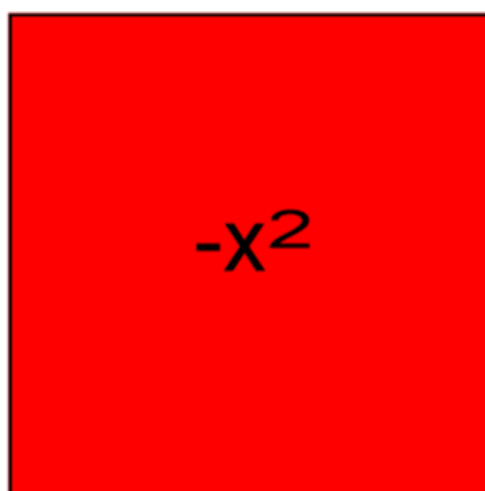
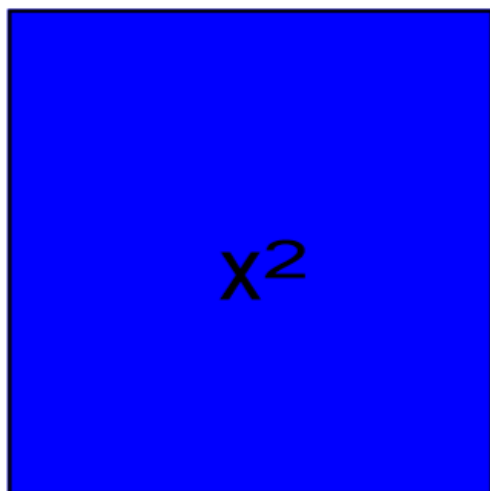
MULTIPLICATION-

DIVISION-

EQUAL-

What piece of information tells you whether or not you are going to create an equation or an expression?

Day 1- Algebra Tiles Activity



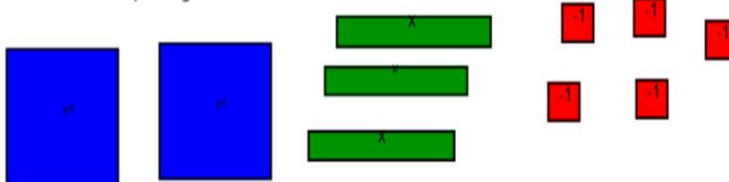
Model Polynomials**Activity 1****Think and Discuss**

1. How do you know when to use red tiles?

Try This

Use algebra tiles to model each polynomial.

1. $2x^2 + 3x - 5$



2. $-4x^2 + 5x - 1$

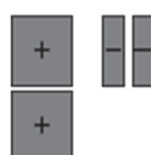
3. $5x^2 - x + 9$

Think and Discuss

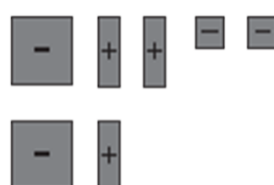
1. How do you know the coefficient of the x^2 term in Activity 2?

Try This

Write a polynomial modeled by each group of algebra tiles.

1. 

2. 

3. 

Day 2

Solving One and Two Step Equations

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Objectives/Goals:

Goal: Solve one and two step equations

Instructional Outcomes:

- 1.) Students will solve one and two step equations using the inverse operations
- 2.) Given an algebraic equation containing addition or subtraction, and multiplication or division, students will be able to solve for the variable

Materials:

- Smart Board
- Worksheets (see attached)

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

Procedure:

- 1.) Warm-up: Students will work on completing the warm-up worksheet as they enter the room. Once they have had about five minutes to solve the problems, we will quickly discuss the answers.
- 2.) Notes: As a class, we will complete the notes and write out the steps taken to solve the equation.
- 3.) Individual Practice: The students will work on the next few examples individually to ensure they understand how to solve them on their own.

- 4.) Partner Practice: The students will work together to complete the practice and show the steps they are taking each time to solve each equation.

Day 2- Solving 1 & 2 step equations

Solving 1 & 2 Step Equations

<u>Work Space</u>	<u>Steps</u>
$3k - 4 = 8$	

Examples:

1. $5 = 3 + d$

2. $-2p + 3 = 11$

3. $x - 4 = 3$

4. $5 + 2x = 7$

5. $-3 - w = 1$

6. $3 = -5f + 13$

<p><u>Work Space</u></p> $\frac{x}{2} + 3 = 1$	<p><u>Steps</u></p>
<p><u>Work Space</u></p> $5 = 2 + 0.75d$	<p><u>Steps</u></p>

1. $2 - \frac{w}{4} = 5$

2. $\frac{2x}{3} = 5$

3. $9y - 7.2 = 4.5$

Day 3

Combining Like Terms and the Distributive Property

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Combining like terms and use the Distributive Property

Instructional Outcomes:

- 1.) Students will use algebra tiles to model the distributive property.
- 2.) Students will solve equations by first using the distributive property.

Materials:

- Smart Board
- Algebra Tiles
- Worksheets (see attached)

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

Procedure:

Warm-up: Students will work on completing the warm-up worksheet as they enter the room. Once they have had about five minutes to solve the problems, we will quickly discuss the answers.

- 1.) Go Over Homework: We will go over last night's homework and discuss the answers. We will ensure that everyone has the correct answers and understands the problems before moving on to combining like terms and learning the Distributive Property.
- 2.) Notes: We will go over the notes as a class. We will discuss the properties and translate them from words to pictures to algebraic equations.
- 3.) Partner Practice: Students will work together at their tables to practice solving equations that require the Distributive Property and combining like terms. Students will have access to algebra tiles to help them solve these problems and to be able to model the equations.

Combing Like Terms and the Distributive Property

like terms: _____

expression: _____

Examples:

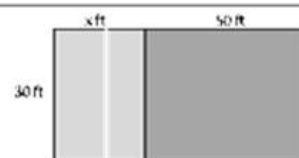
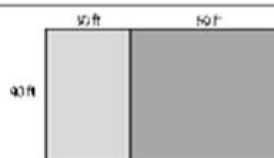
1. $2x - 3 - 4 - 5x$

2. $-4 + 3y - 2x - 6y$

3. $5 + 3y - 2x + 4x - 2y$

The Distributive Property		
Words	Numbers	Algebra

Carlos is trying to calculate the area of his pool. The pool has two areas, one shallow for swimming and one deeper for diving. The diagram below shows the pool. Use the distributive property to write an expression for the area of the pool.



Examples

1.) $5(w+3)$

2.) $4(3x-2)$

3.) $7(2-3y)$

equation: _____

Sometimes you will see these skills in more complex equations. Below are several examples of this.

Examples:

1. $3x - 2x = 10$

2. $3x - 7 - 9x = 25$

3. $34 = 9 - 2x + 5$

4. $4(x + 5) = 40$

5. $2(4x - 7) - 3 = -25$

6. $4x - 3(2x + 8) = -12$

Day 4

Solving Equations with Variables on Both Sides

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Solve equations with variables on both sides

Instructional Outcomes:

1.) Students will solve an equation with a variable on both sides of the equal sign.

Materials:

- Smart Board
- Algebra Tiles
- Note sheet with practice worksheet (see attached below)

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

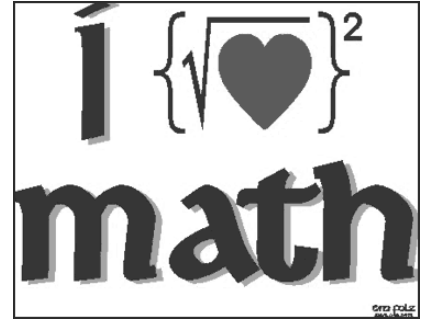
Procedure:

- 1.) Warm-up: Students will work on completing the warm-up worksheet as they enter the room. Once they have had about five minutes to solve the problems, we will quickly discuss the answers.

- 2.) Notes: As a class, we will work through the notes of solving equations with variables on both sides. We will also complete the notes by using algebra tiles to model having variables on both sides of an equation and to see how it will be solved.
- 3.) Partner Practice: Students will work with a partner to complete the practice problems. They will be able to work through the problems together and also have algebra tiles available to help model how to solve equations with variables on both sides.

Name _____

Day 4-Solving Equations Warm-Up

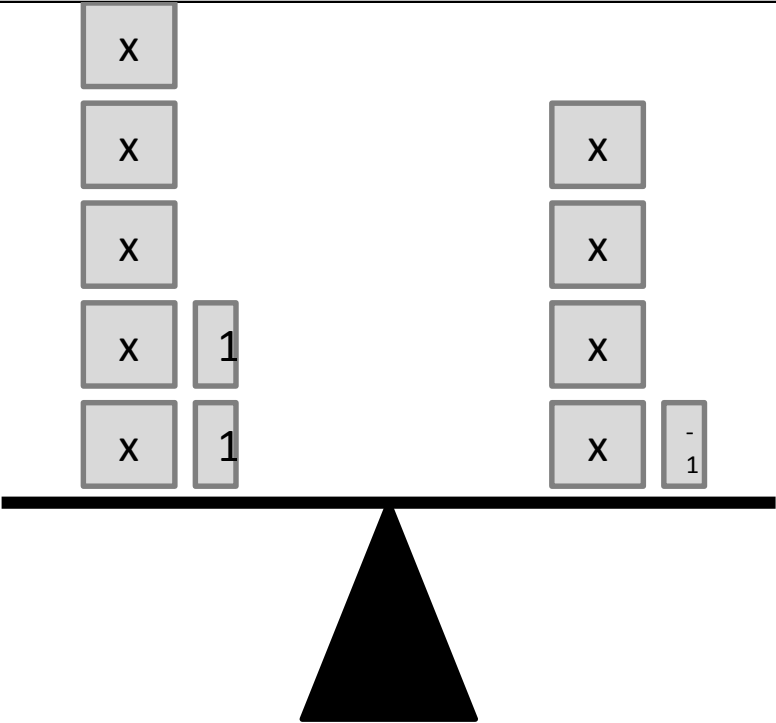
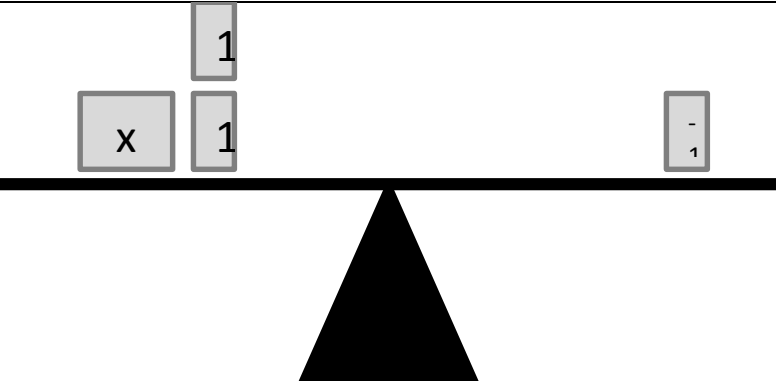


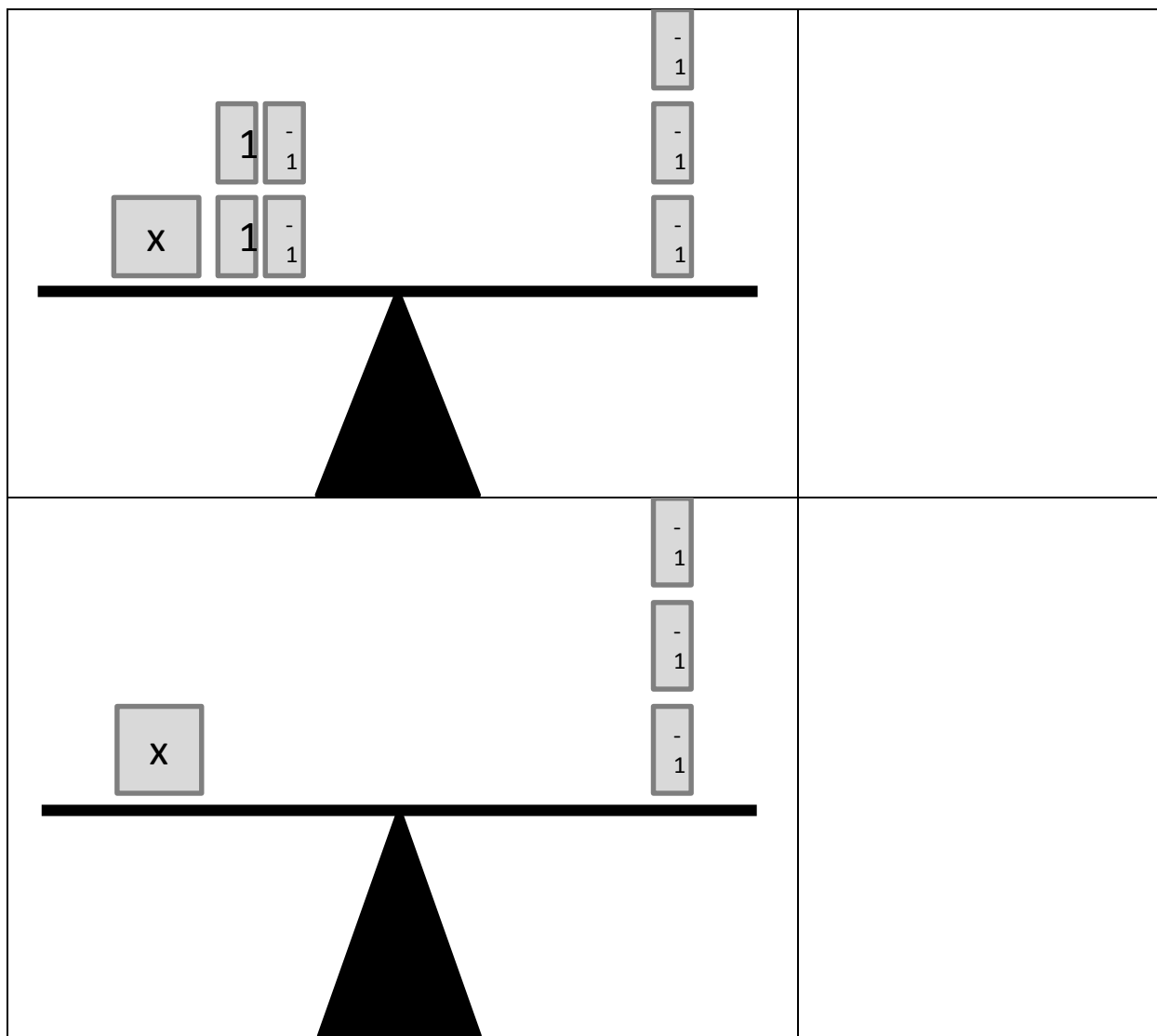
1. Solve for x : $\frac{x}{4} + 7 = 5$

2. Simplify: $6(t - 4) - 5(t + 2)$

3. Solve: $6(2x - 8) + 3 = 15$

Solving Equations with Variables on Both Sides

Visually	Algebraically
	
	



Remember to always get the variable on one side of the equation first,
then work with any constants you may have.

Examples:

1. $7a + 10 = 2a$

2. $11x = 24 + 8x$

3. $8y - 3 = 6y + 17$

4. $7y - 8 = 6y + 1$

5. $15 - \frac{1}{6}n = \frac{1}{6}n - 1$

6. $3 - \frac{2}{9}b = \frac{1}{3}b - 7$

Day 5

Solving Equations-Group Work and Presentation of Results

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Solve complex equations

Instructional Outcomes:

- 1.) Students will practice solving complex equations with groups and share their results with the class.

Materials:

- Class split into assigned groups
- Group sheets-each one having a different problem on it (see attached below)
- Answer sheet for each student to record the findings and presentations of each group

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

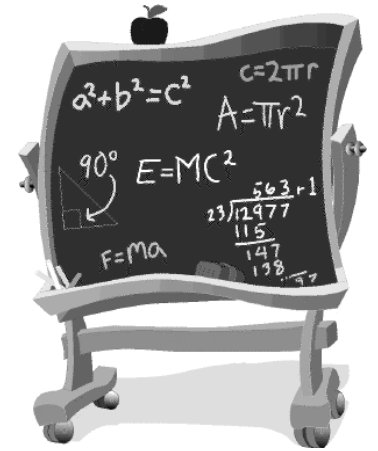
Procedure:

Warm-up: Students will work on completing the warm-up worksheet as they enter the room. Once they have had about five minutes to solve the problems, we will quickly discuss the answers.

- 1.) Group Work: Students will be assigned to a group of about three students. Each group will be assigned one problem. Every group will have a separate problem to do. They will take about 10-15 minutes to do the problem and agree on an answer.
- 2.) Group Presentation: Each group will present the answer they found and the steps they took to get there. It will be the responsibility of the rest of the class to write down the steps and the answer to each problem as each group presents their results. The answer sheets will then be collected.

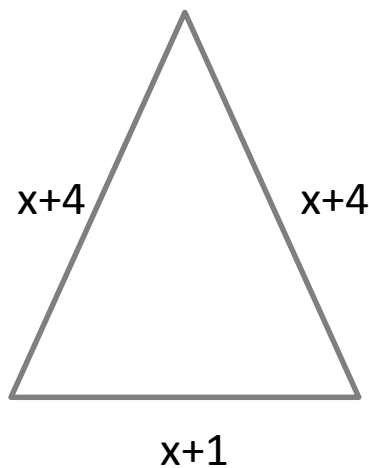
Name _____

Day 5-Solving Equations Warm-Up



1. Solve: $12x - 6 = 10x + 4$

2. In the triangle pictured, the perimeter is 45. Set up and solve an equation to find the value of x .



DAY 5-Solving Equations, group work

Solve the following equations by listening to your peers presentations. Be sure to show all of the steps.

1. $8(3a + 6) = 9(2a - 4)$	2. $4(5 + 2x) - 5 = 3(3x + 7)$	3. $8(t + 2) - 3(t - 4) = 6(t - 7) + 8$
4. $6(2x - 8) + 3 = 15$	5. $-g + 2(3 + g) = -4(g + 1)$	6. $7p - (3p + 4) = -2(2p - 1) + 10$

7. $\frac{1}{2}(-4 + 6x) = \frac{1}{3}x + \frac{2}{3}(x + 9)$	8. $-8 - n = -3(2n - 4)$	9. $2\left(\frac{1}{2}q + 1\right) = -3(2q - 1) + 4(2q + 1)$
10. $-4(2 - y) + 3y = 3(y - 4)$	11. $4(x - 2) = 3(x - 3)$	12. $-5(p + 2) = 2(2p - 15) + p$

Group Members:

Question #1

$$8(3a + 6) = 9(2a - 4)$$

Question #2

$$4(5 + 2x) - 5 = 3(3x + 7)$$

Question #3

$$8(t + 2) - 3(t - 4) = 6(t - 7) + 8$$

Question #4

$$6(2x - 8) + 3 = 15$$

Question #5

$$-g + 2(3 + g) = -4(g + 1)$$

Question #6

$$7p - (3p + 4) = -2(2p - 1) + 10$$

Question #7

$$\frac{1}{2}(-4 + 6x) = \frac{1}{3}x + \frac{2}{3}(x + 9)$$

Question #8

$$-8 - n = -3(2n - 4)$$

Question #9

$$2\left(\frac{1}{2}q + 1\right) = -3(2q - 1) + 4(2q + 1)$$

Question #10

$$-4(2 - y) + 3y = 3(y - 4)$$

Question #11

$$4(x - 2) = 3(x - 3)$$

Question #12

$$-5(p + 2) = 2(2p - 15) + p$$

For this activity, each question was put on one page. Then pages were then passed out to the different groups, so each group had one or two problems with one page per problem. After the groups had an allotted amount of time to work on the problems, the group would share their findings with the whole class.

Day 6

Identifying Solutions of an Equation

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Identify how many solutions an equation will have

Instructional Outcomes:

1.) Students will be able to determine the number of solutions an equation will have.

Materials:

- Smart Board
- Worksheet (see attached)

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

Procedure:

- 1.) Notes: We will discuss how to determine how many solutions an equation has and what the different number of solutions mean.
- 2.) Partner Practice: Students will work together on a specific set of problems and practice evaluating how many solutions each equation has and what this means.

Name _____

Day 6-Solving Solutions warm-up

1. Simplify: $3x - 2(4x - 5) + 8x - 12$



Number of Solutions of an Equation

null set: _____

Symbols: _____

identity: _____

	Null Set	One Solution	Identity
Words			
Algebra Representation			
Example			
Cues to look for			

Solve the following equations to determine the number of solutions the equation has.

1. $2(x - 1) = 2x + 2$

2. $-5(3m + 6) = -3(4m - 2)$

3. $12(x + 3) = 4(2x + 9) + 4x$

4. $2h + 3 = 2h + 5$

5. $3y + 5 = 3y - 2 + 7$

6. $-8(w - 6) = 32$

7. $8(3a + 6) = 9(2a - 4)$

8. $8(c - 9) = 6(2c - 12) - 4c$

9. $-10y + 18 = -3(5y - 7) + 5y$

For each equation, work with your partner to find the solution. If you get stuck on one, make an educated guess as to the solution and move on to the next so you are sure to get all three completed in the given time.

$2x + 4 = 2(x + 6)$

$4(x - 3) = 6x + 8$

$3(x - 2) = 3x - 6$

Day 7

Review Day

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Review solving equations

Instructional Outcomes:

1.) Students will review solving equations.

Materials:

- Smart Board
- Review Worksheets (attached below)

Anticipatory Set:

Warm-up: Written on the front board

~ Write homework in your plan book

~Have last night's homework out on your desk

Procedure:

- 1.) Classwork: Students will work with a partner, to work through the review packet. They will be allowed to discuss the math and help one another.
- 2.) Homework: Students will be expected to complete the review packet for homework and to study for the test.

Review Worksheet for Unit Test

1. $27 = 12 - 5x$

2. $3(x + 1) = 1 + 3x + 2$

3. $12(x + 3) = 4(2x + 9) + 4x$

4. $-27 - 15 = -12h + 4h - 3 + 15 - h$

5. $4(2p + 5) - 6 + 2p = 44$

6. $2q - 14 = 4(q + 5)$

7. $-2y - 3y + 8 = 8 - 5y - 11$

8. $3x - 2x + 17 - 4x = 32$

9. $\frac{-2x}{3} + 11 = -5$

10. The perimeter of a rectangle is $8(2x + 1)$ inches. If the length of the sides of the rectangle are $3x + 4$ inches and $4x + 3$ inches, what is the length of each side of the rectangle? (Hint: the formula for the perimeter of a rectangle is $P = 2l + 2w$)

11. Number the steps in the order needed to solve the question.

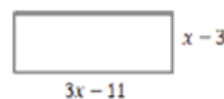
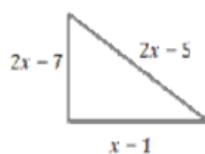
$$-\frac{2}{3}(a + 3) = \frac{5}{3}a - 19$$

- _____ Add $\frac{2}{3}a$ to each side.
 _____ Add 19 to each side.
 _____ Multiple a and 3 by $-\frac{2}{3}$.
 _____ Multiply each side by $\frac{3}{7}$.

Now, solve the equation using the above steps.

$$-\frac{2}{3}(a + 3) = \frac{5}{3}a - 19$$

12. Elin wants to fence in two different garden plots in her back yard with two rolls of fencing that are the same length. Write and solve an equation to find the value of x so that the figures below have the same perimeter.



Day 8

Assessment: Solving Equations

NYS Learning Standards:

8.EE

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Objectives/Goals:

Goal: Show me what you have learned about solving equations

Instructional Outcomes:

1.) Students will complete the Unit Assessment

Materials:

- Unit Assessment

Anticipatory Set:

Warm-up: Written on the front board

~Take out a pencil and put all other materials away.

Procedure:

1.) Unit Assessment

Students will be given the entire class time to complete the module assessment.

Test - Solving Equations

Name: _____ Period: _____ Date: _____

Part I: Multiple-Choice (1 point each)

_____ 1.) Solve: $10 + 3y = 1$

(A) $y = -9$ (C) $y = 3$

(B) $y = -3$ (D) $y = 9$

_____ 2.) Solve: $\frac{4}{7}w = 16$

(A) $w = 4$ (C) $w = 14$

(B) $w = 28$ (D) $w = 112$

_____ 3.) Solve: $\frac{x}{2} - 5 = -3$

(A) $x = 4$ (C) $x = -4$

(B) $x = 1$ (D) $x = -16$

_____ 4.) Solve: $4q - 2 = 22 - 8q$

(A) null set (C) $q = 6$

(B) $q = -6$ (D) $q = 2$

_____ 5.) Solve: $-3(p + 2) = -30$

(A) $p = -\frac{32}{3}$ (C) $p = -8$

(B) $p = 8$ (D) $p = \frac{32}{3}$

_____ 6.) Solve: $4(2 - k) = 2(k + 7)$

- | | |
|--------------|--------------|
| (A) $k = -3$ | (C) $k = 3$ |
| (B) $k = -1$ | (D) $k = 11$ |

_____ 7.) Simplify: $5 + 4(2 + 3x)$

- | | |
|---------------|----------------|
| (A) $3x + 11$ | (C) $12x + 13$ |
| (B) $3x + 13$ | (D) $12x + 8$ |

_____ 8.) Solve: $2(x + 3) = 2x + 6$

- | | |
|--------------|--------------|
| (A) $x = 6$ | (C) $x = -3$ |
| (B) null set | (D) identity |

_____ 9.) Solve: $-3y - 4y + 9 = 9 - 7y - 12$

- | | |
|--------------|--------------------------|
| (A) identity | (C) null set |
| (B) $x = -2$ | (D) $x = -\frac{12}{24}$ |

_____ 10.) Solve: $p + 13 = 8$

- | | |
|--------------|---------------|
| (A) $p = -5$ | (C) $p = 21$ |
| (B) $p = 5$ | (D) $p = 104$ |

Part II – Open Response (2 points each)

11.) Solve: $5.6x + 22.1 = -2.1 + 7.6x$

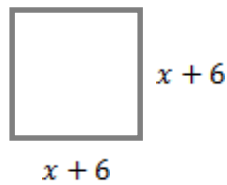
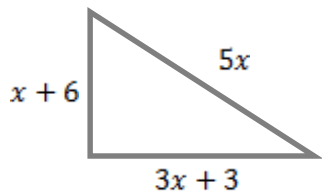
12.) Solve: $12(x + 3) = 8x + 36 + 4x$

13.) Solve: $\frac{x + 3}{2} = 6$

Part III – Extended Response (3 points each)

14.) Solve: $2(3x + 1) + 2x = 8x + 4$

- 15.) Write and solve an equation to find the value of x that makes the perimeter of the figures below equal.



16.) Solve: $8(x + 2) - 3(x - 4) = 6(x - 7) + 8$

***Bonus* (1 point)**

Solve: $\frac{3x}{4} + \frac{16}{3} = -2(x + 13)$

Chapter 4

Validity of Curriculum Project

The validity of this curriculum project was assessed by veteran teachers of eighth grade students, as the unit plan was implemented in the classroom in October, 2015. The classroom was composed of a total of 62 total students in an urban school district, divided into three sections. The two other co-teachers who were asked to assess the unit plan and use their expertise to critique and reflect on the unit that was created. The teachers were asked to do this by answering questions provided in a questionnaire and also informal discussions about the unit plan and lessons. They were asked:

- 1.) Based on the Common Core State Learning Standards, was this unit plan appropriately aligned to the standards?
- 2.) What would you consider to be strengths of this unit plan?
- 3.) What suggestions or changes would you make to this unit plan?
- 4.) Discuss any other comments you may have and how you feel it worked with the students, how engaged they were, and if the learning targets were met.

With the help of the answers provided by these questionnaires and also the discussion that were had with the co-teachers, the author of the curriculum project was able to measure the validity.

Chapter 5

Summary

Further Opportunities with the Common Core State Standards

The move from the NCTM standards to the Common Core State Standards allows educators an opportunity to restructure their teaching in the classroom. This curriculum project was created as a resource to aid in the shift from NCTM to CCSS. One of the main opportunities of the CCSS was for “teacher preparation and professional development to become less generic and more focused on the mathematics taught at a given grade level” (McCallum, 2012). As stated by McCallum, the Common Core is based on progressions that start with a trickle before they grow into a full flow (2012). This allows educators the opportunity to re-design their curriculum and classroom teachings and to move away from the “mile wide, inch deep” curriculum. As McCallum states, the fundamental aspects of the standards are focus, coherence, and rigor. For teachers having common standards creates a focus rather than the various demands put in place by standards. Having common standards can also be a useful tool for teachers, to share their resources for implementation. Not only will efficiency be increased but also having a common set of standards will allow students to move to a different school and not have their education interrupted or be left with any gaps.

Curriculum Project Results and Findings

Two teachers with many years of experience agreed to complete the questionnaire and supply their thoughts on the unit plan after analyzing and implementing the lessons. The two teachers work in the city, one is in a charter school. They both teach a group of eighth grade math students. After discussions and receiving feedback, it is clear that both veteran teachers would use this unit plan in their classroom and agree that it is an effective teaching tool.

The author of this Unit Plan was able to use the lessons that were created in their own eighth grade classroom. As a reflection, the four questions from the questionnaire were answered below:

1.) Based on the Common Core State Learning Standards, was this unit plan appropriately aligned to the standards?

The Unit Plan was aligned to the Common Core State Learning Standards as students developed and displayed a deeper understanding of Solving Equations. Students were not only able to solve for a variable in one or two steps, but also solve more complex equations. This is a key component of the curriculum and used in mathematics throughout many levels.

2.) What would you consider to be strengths of this unit plan?

This Unit Plan takes into consideration how to engage middle school students while also addressing the standards. One of the most enjoyable activities for the students was the algebra tiles activity. Many students liked the hands on experience and using the manipulatives to help model how to solve equations.

3.) What suggestions or changes would you make to this unit plan?

The idea of using the manipulatives more would help the students. However, adding more real world problems or problems that the students can relate to would be beneficial for the students.

4.) Discuss any other comments you may have and how you feel it worked with the students, how engaged they were, and if the learning targets were met.

This Unit Plan worked well with the students but there is always room for improvement. They were engaged, especially when having a hands on activity. The students would also

be more engaged if there were more real life scenarios to solve for or problems the students could relate to. The learning targets of this Unit Plan were met.

In conclusion, the goal of this curriculum project was to design a unit plan for Mathematics 8. This was created to be used as a resource as classrooms go through the instructional shift from NCTM to the Common Core State Standards. The implementation and constructive criticism of this curriculum project show that the goal was achieved.

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Resources

The Common Core State Standard Initiative. <http://www.corestandards.org>

Appendix A

Unit Plan Answer Key

The answer keys to all worksheets, warm-ups, tests, and activities are included.

Solving 1 & 2 Step Equations

Work Space	Steps
$\begin{array}{rcl} 3K - 4 & = & 8 \\ +4 & & +4 \\ \hline 3K & = & 12 \\ \div 3 & & \div 3 \\ \hline K & = & 4 \end{array}$	<p>1.) Eliminate addition/subtraction.</p> <p>2.) Eliminate multiplication/division.</p>

Examples:

1. $5 = 8 + d$

$$\begin{array}{rcl} 5 & = & 8 + d \\ -8 & & -8 \\ \hline -3 & = & d \end{array}$$

2. $-2p + 3 = 11$

$$\begin{array}{rcl} -2p + 3 & = & 11 \\ -3 & & -3 \\ \hline -2p & = & 8 \\ \div -2 & & \div -2 \\ \hline p & = & -4 \end{array}$$

3. $x + 4 = 3$

$$\begin{array}{rcl} x + 4 & = & 3 \\ -4 & & -4 \\ \hline x & = & -1 \end{array}$$

4. $5 + 2x = 7$

$$\begin{array}{rcl} 5 + 2x & = & 7 \\ -5 & & -5 \\ \hline 2x & = & 2 \\ \div 2 & & \div 2 \\ \hline x & = & 1 \end{array}$$

5. $-3 - w = 1$

$$\begin{array}{rcl} -3 - w & = & 1 \\ +3 & & +3 \\ \hline -w & = & 4 \\ \div -1 & & \div -1 \\ \hline w & = & -4 \end{array}$$

6. $3 = -5f + 13$

$$\begin{array}{rcl} 3 & = & -5f + 13 \\ -13 & & -13 \\ \hline -10 & = & -5f \\ \div -5 & & \div -5 \\ \hline 2 & = & f \end{array}$$

<p><u>Work Space</u></p> $\begin{array}{r l} \textcircled{x} + 3 = 1 \\ -3 \\ \hline \textcircled{x} = -2 \end{array}$ $\begin{array}{r l} 2x = -2(2) \\ \hline x = -4 \end{array}$	<p><u>Steps</u></p> <p>1.) Eliminate addition/subtraction.</p> <p>2.) Eliminate multiplication/division.</p>
<p><u>Work Space</u></p> $\begin{array}{r l} 5 = 2 + 0.75d \\ -2 \\ \hline 3 = 0.75d \\ \hline 0.75 \quad 0.75 \\ \hline 4 = d \end{array}$	<p><u>Steps</u></p> <p>1.) Eliminate addition/subtraction.</p> <p>2.) Eliminate division/multiplication.</p>

$$\begin{array}{r|l} 1. \quad 2 - \frac{w}{4} = 5 \\ -2 \\ \hline -\frac{w}{4} = 3 \\ \hline -w = 3(-4) \\ \hline w = -12 \end{array}$$

$$\begin{array}{r|l} 2. \quad \frac{3x}{2} = 5(\frac{3}{2}) \\ \hline x = \frac{15}{2} \end{array}$$

$$\begin{array}{r|l} 3. \quad \frac{y}{9} + 7.2 = 4.5 \\ -7.2 \\ \hline \frac{y}{9} = -2.7 \\ \hline y = -2.7(9) \\ \hline y = -24.3 \end{array}$$

Combing Like Terms and the Distributive Property

like terms: terms in which all parts (variables and exponents) are the same, except numerical coefficients

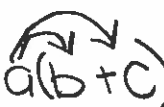


expression: a combination of numbers, symbols, and operations to represent a quantity (no equal sign).

Examples:

1. $(2x - 3) - 4 - 5a$
 $-3x - 7$

2. $(-4 + 3y) - 2x - 6y$
 $-4 - 3y - 2x$

3. $(5 + 3y) - 2x + 4x - 2y$
 $5 + y + 2x$

The Distributive Property		
Words	Numbers	Algebra
Multiplying a sum by a number, gives the same result as multiplying each addend by the number.	 $a(b+c)$	$= ab + ac$
<p>Carlos is trying to calculate the area of his pool. The pool has two areas, one shallow for swimming and one deeper for diving. The diagram below shows the pool. Use the distributive property to write an expression for the area of the pool.</p>		

Examples

1.) $5(w+3)$

$5w + 15$

2.) $4(3x-2)$

$12x - 8$

3.) $7(2-3y)$

$14 - 21y$

equation: a mathematical statement that represents the equality of two expressions. (has an equal sign)

Sometimes you will see these skills in more complex equations. Below are several examples of this.

Examples:

1. $3x - 2x = 10$
 $x = 10$

2. $8x - 7 - 9x = 25$

$$\begin{array}{r} -6x + 7 = 25 \\ +7 \\ \hline -6x = 18 \\ \div -6 \\ \hline x = -3 \end{array}$$

3. $34 = 9 - 2x + 5$

$$\begin{array}{r} 34 = 14 - 2x \\ -14 \\ \hline 20 = -2x \\ \div -2 \\ \hline -10 = x \end{array}$$

4. $4(x + 5) = 40$

$$\begin{array}{r} 4x + 20 = 40 \\ -20 \\ \hline 4x = 20 \\ \div 4 \\ \hline x = 5 \end{array}$$

5. $2(4x - 7) - 3 = -25$

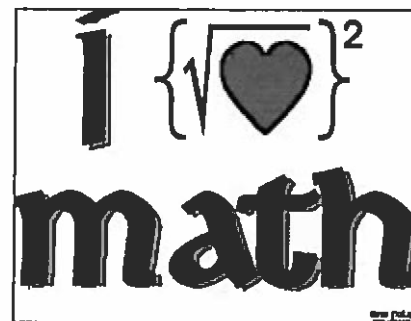
$$\begin{array}{r} 8x - 14 - 3 = -25 \\ 8x - 17 = -25 \\ +17 \\ \hline 8x = -8 \\ \div 8 \\ \hline x = -1 \end{array}$$

6. $4x - 3(2x + 8) = -12$

$$\begin{array}{r} 4x - 6x - 24 = -12 \\ -2x - 24 = -12 \\ +24 \\ \hline -2x = 12 \\ \div -2 \\ \hline x = -6 \end{array}$$

Name _____

Math 8: Warm-up #3



1. Solve for x:

$$\frac{x}{4} + 7 = 5$$
$$\frac{x}{4} = -2$$
$$x = -8$$

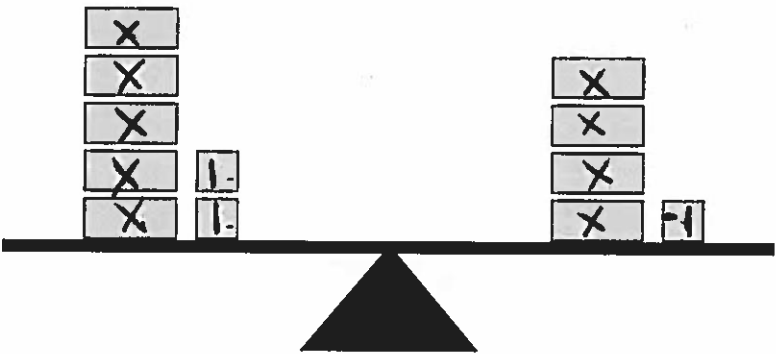
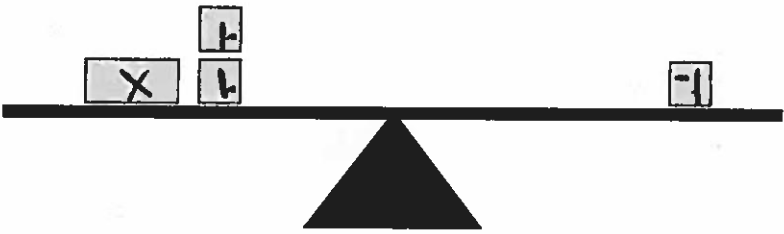
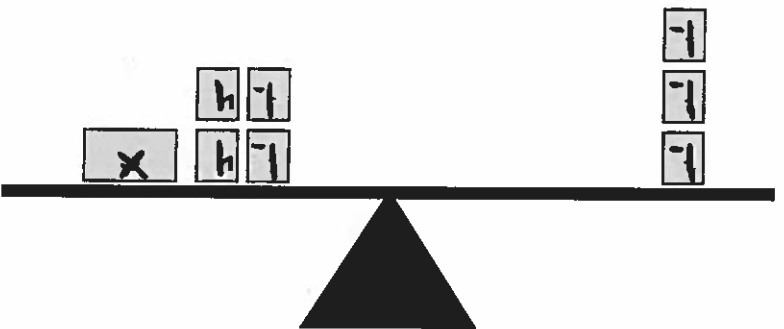
2. Simplify: $6(t - 4) - 5(t + 2)$

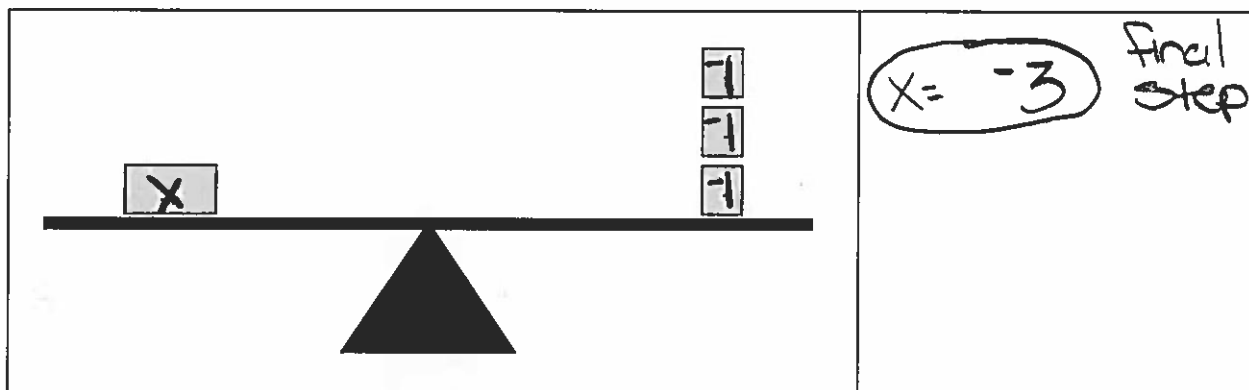
$$6t - 24 - 5t - 10$$
$$t - 34$$

3. Solve: $6(2x - 8) + 3 = 15$

$$12x - 48 + 3 = 15$$
$$12x - 45 = 15$$
$$+45 \quad +45$$
$$12x = 60$$
$$\div 12 \quad \div 12$$
$$x = 5$$

Solving Equations with Variables on Both Sides

Visually	Algebraically
	$5x + 2 = 4x - 1$ $\begin{array}{r} 5x + 2 = 4x - 1 \\ -4x \quad -4x \\ \hline x + 2 = -1 \\ -2 \quad -2 \\ \hline x = -3 \end{array}$ <p style="text-align: right;">next step</p>
	$x + 2 = -1$ $\begin{array}{r} x + 2 = -1 \\ -2 \quad -2 \\ \hline x = -3 \end{array}$ <p style="text-align: right;">next step</p>
	$x + 2 - 2 = -3$ $\begin{array}{r} x + 2 - 2 = -3 \\ \hline x = -3 \end{array}$ <p style="text-align: right;">next step</p>



Remember to always get the variable on one side of the equation first,
then work with any constants you may have.

Examples:

$$1. \quad \cancel{7}a + 10 = 2a$$

$$\begin{array}{r} \cancel{7}a \\ -7a \\ \hline 10 = \cancel{2}a \\ -2a \\ \hline \end{array}$$

$$\textcircled{-2 = 9}$$

$$2. \quad 11x = 24 + \cancel{8}x$$

$$\begin{array}{r} 11x \\ -8x \\ \hline 3x = 24 \\ \div 3 \\ \hline \end{array}$$

$$\textcircled{x = 8}$$

$$3. \quad 8y - 3 = \cancel{6}y + 17$$

$$\begin{array}{r} 8y \\ -6y \\ \hline 2y - 3 = 17 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\begin{array}{r} 2y = 20 \\ \div 2 \\ \hline \end{array}$$

$$\textcircled{y = 10}$$

$$4. 7y - 8 = 6y + 1$$

$$\begin{array}{r} \cancel{6y} - \cancel{6y} \\ \hline 7y - 8 = 1 \\ +8 \quad +8 \\ \hline y = 9 \end{array}$$

$$5. 15 - \frac{1}{6}n = \frac{1}{6}n - 1$$

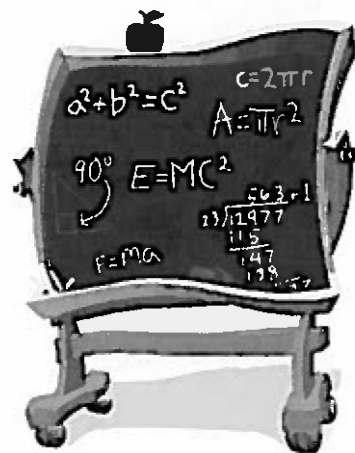
$$\begin{array}{r} \cancel{+1} \quad \cancel{+1} \\ \hline 6n \quad 6n \\ \hline 15 = \frac{2}{6}n \\ +1 \quad \cancel{+1} \\ \hline (3) 16 = \frac{2}{3}(3) \\ 48 = n \end{array}$$

$$6. 3 - \frac{2}{9}b = \frac{1}{3}b - 7$$

$$\begin{array}{r} \cancel{+3} \quad \cancel{+3} \\ \hline 9b \quad 9b \\ \hline 3 = \frac{5}{9}b + 7 \\ +7 \quad +7 \\ \hline (\frac{9}{5}) 10 = \frac{5}{9}b (\frac{9}{5}) \\ 18 = b \end{array}$$

Name _____

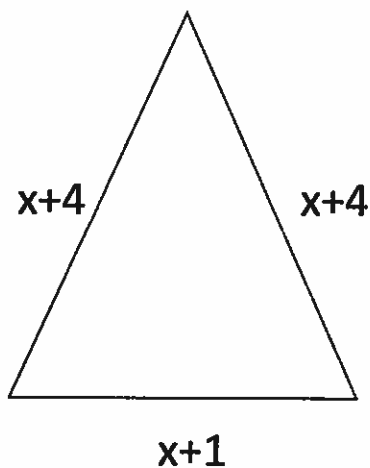
Math 8: Warm-up #4



1. Solve: $12x - 6 = 10x + 4$

$$\begin{array}{r} \underline{-10x} \quad \underline{-10x} \\ 2x - 6 = 4 \\ \underline{+6} \quad \underline{+6} \\ 2x = 10 \\ \underline{\div 2} \quad \underline{\div 2} \\ x = 5 \end{array}$$

2. In the triangle pictured, the perimeter is 45. Set up and solve an equation to find the value of x .



$$\begin{array}{r} \textcircled{x+4} + \textcircled{x+4} + \textcircled{x+1} = 45 \\ 3x + 9 = 45 \\ \underline{-9} \quad \underline{-9} \\ 3x = 36 \\ \underline{\div 3} \quad \underline{\div 3} \\ x = 12 \end{array}$$

Student answer sheet.

Answers to all 12 groups.

Solve the following equations by listening to your peers presentations. Be sure to show all of the steps.

1. $8(3a + 6) = 9(2a - 4)$

$$\begin{array}{r} 24a + 48 = 18a - 36 \\ -18a \quad -18a \\ \hline 6a + 48 = -36 \end{array}$$

$$\begin{array}{r} 6a + 48 = -36 \\ -48 \quad -48 \\ \hline 6a = -84 \end{array}$$

$$\frac{6a}{6} = \frac{-84}{6}$$

$$a = -14$$

2. $4(5 + 2x) - 5 = 3(3x + 7)$

$$20 + 8x - 5 = 9x + 21$$

$$\begin{array}{r} 15 + 8x = 9x + 21 \\ -8x \quad -8x \\ \hline 15 = x + 21 \end{array}$$

$$\begin{array}{r} 15 = x + 21 \\ -21 \quad -21 \\ \hline -6 = x \end{array}$$

$$-6 = x$$

3. $8(t + 2) - 3(t - 4) = 6(t - 7) + 8$

$$8t + 16 - 3t + 12 = 6t - 42 + 8$$

$$\begin{array}{r} 5t + 28 = 6t - 34 \\ -5t \quad -5t \\ \hline 28 = t - 34 \end{array}$$

$$\begin{array}{r} 28 = t - 34 \\ +34 \quad +34 \\ \hline 62 = t \end{array}$$

$$62 = t$$

4. $6(2x - 8) + 3 = 15$

$$12x - 48 + 3 = 15$$

$$\begin{array}{r} 12x - 45 = 15 \\ +45 \quad +45 \\ \hline 12x = 60 \end{array}$$

$$\frac{12x}{12} = \frac{60}{12}$$

$$x = 5$$

5. $-g + 2(3 + g) = -4(g + 1)$

$$-g + 6 + 2g = -4g - 4$$

$$\begin{array}{r} g + 6 = -4g - 4 \\ +4g \quad +4g \\ \hline 5g + 6 = -4 \end{array}$$

$$\begin{array}{r} 5g + 6 = -4 \\ -6 \quad -6 \\ \hline 5g = -10 \end{array}$$

$$\frac{5g}{5} = \frac{-10}{5}$$

$$g = -2$$

6. $7p - (3p + 4) = -2(2p - 1) + 10$

$$7p - 3p - 4 = -4p + 2 + 10$$

$$\begin{array}{r} 4p - 4 = -4p + 12 \\ +4p \quad +4p \\ \hline 8p - 4 = 12 \end{array}$$

$$\begin{array}{r} 8p - 4 = 12 \\ +4 \quad +4 \\ \hline 8p = 16 \end{array}$$

$$\frac{8p}{8} = \frac{16}{8}$$

$$p = 2$$

$$7. \frac{1}{2}(-4 + 6x) = \frac{1}{3}x + \frac{2}{3}(x + 9)$$

$$-2 + 3x = \frac{1}{3}x + \frac{2}{3}x + 6$$

$$-2 + 3x = \frac{1}{3}x + 6$$

$$2 + 2x = 6$$

$$2x = 4$$

$$x = 4$$

$$8. -8 - n = -3(2n - 4)$$

$$-8 - n = -6n + 12$$

$$-8 = -5n + 12$$

$$-20 = -5n$$

$$4 = n$$

$$9. 2\left(\frac{1}{2}q + 1\right) = -3(2q - 1) + 4(2q + 1)$$

$$q + 2 = -6q + 3 + 8q + 4$$

$$q + 2 = 2q + 7$$

$$2 = q + 7$$

$$-5 = q$$

$$10. -4(2 - y) + 3y = 3(y - 4)$$

$$-8 + 4y + 3y = 3y - 12$$

$$-8 + 7y = 3y - 12$$

$$-8 + 4y = -12$$

$$4y = -4$$

$$y = -1$$

$$11. 4(x - 2) = 3(x - 3)$$

$$4x - 8 = 3x - 9$$

$$x + 8 = -9$$

$$x = -1$$

$$12. -5(p + 2) = 2(2p - 15) + p$$

$$-5p - 10 = 4p - 30 + p$$

$$-5p - 10 = 5p - 30$$

$$-10 = 10p - 30$$

$$20 = 10p$$

$$2 = p$$

Name _____

Math 8: Warm-up #5



1. Simplify: $3x - 2(4x - 5) + 8x - 12$

$$\begin{array}{r} \textcircled{3x} - \textcircled{8x} + 10 + \textcircled{8x} - 12 \\ \hline 3x - 2 \end{array}$$

2. Solve: $4x + 2(x - 5) = 5x - 3(x + 10)$

$$\textcircled{4x} + \textcircled{2x} - 10 = \textcircled{5x} - \textcircled{3x} - 30$$

$$\begin{array}{r} 12x - 10 = 2x - 30 \\ \underline{-2x} \quad \underline{-2x} \end{array}$$

$$\begin{array}{r} 10x + 10 = -30 \\ \underline{+10} \quad \underline{+10} \end{array}$$

$$\begin{array}{r} 10x = -20 \\ \underline{10} \quad \underline{10} \end{array}$$

$$\textcircled{x = -2}$$

Number of Solutions of an Equation

null set: empty set or no solution

Symbols: $\emptyset, \{ \}$

identity: infinitely many solutions

	Null Set	One Solution	Identity
Words	No solution	Equation has One solution	Infinitely many solutions
Algebra Representation	\emptyset	$x = \underline{\text{only one}}$	∞
Example	$-2x - 5 = -2x + 7$	$5x + 6 = 4x + 10$ $2x + 7 = 7 + 2x$	
Cues to look for	Same coefficients different Constant	different Coefficients different Constants	Same coefficients Same constants

Solve the following equations to determine the number of solutions the equation has.

$$1. \quad 2x - 1 = 2x + 2$$

$$\begin{array}{r} 2x - 1 = 2x + 2 \\ -2x \quad -2x \\ \hline -1 = 2 \end{array}$$

No Solution

$$2. \quad -5(3m + 6) = -3(4m - 2)$$

$$\begin{array}{r} -15m - 30 = -12m + 6 \\ +12m \quad +12m \\ \hline -3m - 30 = 6 \\ +30 \quad +30 \\ \hline -3m = 36 \\ \div -3 \quad \div -3 \\ \hline m = -12 \end{array}$$

One Solution

$$3. \quad 12(x + 3) = 4(2x + 9) + 4x$$

$$\begin{array}{r} 12x + 36 = 8x + 36 + 4x \\ 12x + 36 = 12x + 36 \end{array}$$

Infinitely Many Solutions

$$4. \quad 2h + 3 = 2h + 5$$

$$\begin{array}{r} 2h + 3 = 2h + 5 \\ -2h \quad -2h \\ \hline 3 = 5 \end{array}$$

No Solution

$$5. \quad 3y + 5 = 3y - 2 + 7$$

$$\begin{array}{r} 3y + 5 = 3y + 5 \end{array}$$

Infinitely Many Solutions

$$6. \quad -8(w - 6) = 32$$

$$\begin{array}{r} -8w + 48 = 32 \\ -48 \quad -48 \\ \hline -8w = -16 \\ \div -8 \quad \div -8 \\ \hline w = 2 \end{array}$$

One Solution

$$7. \quad 8(3a + 6) = 9(2a - 4)$$

$$\begin{array}{r} 24a + 48 = 18a - 36 \\ -18a \quad -18a \\ \hline 6a + 48 = -36 \\ -48 \quad -48 \\ \hline 6a = -84 \\ \div 6 \quad \div 6 \\ \hline a = -14 \end{array}$$

One Solution

$$8. \quad 8(c - 9) = 6(2c - 12) - 4c$$

$$\begin{array}{r} 8c - 72 = 12c - 72 - 4c \\ 8c - 72 = 8c - 72 \end{array}$$

Infinitely Many Solutions

$$9. \quad -10y + 18 = -3(5y - 7) + 5y$$

$$\begin{array}{r} -10y + 18 = -15y + 21 + 5y \\ +15y \quad +15y \\ \hline 18 = 21 \end{array}$$

No Solution

For each equation, work with your partner to find the solution. If you get stuck on one, make an educated guess as to the solution and move on to the next so you are sure to get all three completed in the given time.

$$2x + 4 = 2(x + 6)$$

$$\begin{array}{r} 2x + 4 = 2x + 12 \\ -2x \quad -2x \\ \hline 4 = 12 \end{array}$$

No Solution

$$4(x - 3) = 6x + 8$$

$$\begin{array}{r} 4x - 12 = 6x + 8 \\ -4x \quad -4x \\ \hline -12 = 2x + 8 \\ -8 \quad -8 \\ \hline -20 = 2x \\ \div 2 \quad \div 2 \\ \hline x = -10 \end{array}$$

One Solution

$$3(x - 2) = 3x - 6$$

$$\begin{array}{r} 3x - 6 = 3x - 6 \end{array}$$

Infinitely Many Solutions

Review Worksheet

$$\begin{array}{r}
 1. \quad 27 = 12 - 5x \\
 \underline{-12} \quad \underline{-12} \\
 15 = -5x \\
 \underline{-5} \quad \underline{-5} \\
 -3 = -x \\
 \hline
 -3 = x
 \end{array}$$

$$\begin{array}{l}
 2. \quad 3(x+1) = 1 + 3x + 2 \\
 3x + 3 = 3 + 3x
 \end{array}$$

Infinitely
Many
Solutions

$$\begin{array}{l}
 3. \quad 12(x+3) = 4(2x+9) + 4x \\
 12x + 36 = 8x + 36 + 4x \\
 12x + 36 = 12x + 36
 \end{array}$$

Infinitely
Many
Solutions

$$\begin{array}{r}
 4. \quad -27 - 15 = -12h + 4h - 3 + 15 - h \\
 -42 = -8h - 3 + 15 - h \\
 \underline{-12} \quad \underline{-12} \quad \underline{-12} \\
 -54 = -9h + 12 \\
 \underline{-9} \quad \underline{-9} \quad \underline{-9} \\
 -66 = -9h \\
 \hline
 6 = h
 \end{array}$$

$$\begin{array}{l}
 5. \quad 4(2p+5) - 6 + 2p = 44 \\
 8p + 20 - 6 + 2p = 44 \\
 10p + 14 = 44 \\
 \underline{-14} \quad \underline{-14} \\
 10p = 30 \\
 \underline{10} \quad \underline{10} \\
 p = 3
 \end{array}$$

$$\begin{array}{l}
 6. \quad 2q - 14 = 4(q+5) \\
 2q - 14 = 4q + 20 \\
 \underline{-2q} \quad \underline{-2q} \\
 -14 = 2q + 20 \\
 \underline{-20} \quad \underline{-20} \\
 -34 = 2q \\
 \underline{-2} \quad \underline{-2} \\
 -17 = q
 \end{array}$$

7. $-2y - 3y + 8 = 8 - 5y - 11$

$$\begin{array}{r} -5y + 8 = -5y - 3 \\ +2y \quad \quad +2y \\ \hline 8 = -3 \end{array}$$

No
Solution

8. $3x - 2x + 17 - 4x = 32$

$$\begin{array}{r} -3x + 17 = 32 \\ -17 \quad -17 \\ \hline -3x = 15 \\ \div -3 \quad \div -3 \\ \hline x = -5 \end{array}$$

9. $\frac{-3}{2} + \frac{11}{3} = -5$

$$\begin{array}{r} (-\frac{3}{2}) + \frac{11}{3} = -5 \\ \times \frac{6}{6} \\ \hline (-\frac{9}{2}) + \frac{22}{3} = -5 \\ \hline x = 24 \end{array}$$

10. The perimeter of a rectangle is $8(2x + 1)$ inches. If the length of the sides of the rectangle are $3x + 4$ inches and $4x + 3$ inches, what is the length of each side of the rectangle? (Hint: the formula for the perimeter of a rectangle is $P = 2l + 2w$)

$$\begin{array}{l} 8(2x + 1) = 2(3x + 4) + 2(4x + 3) \\ 16x + 8 = 6x + 8 + 8x + 6 \\ 16x + 8 = 14x + 14 \\ -14x \quad -14x \\ \hline 2x + 8 = 14 \\ -8 \quad -8 \\ \hline 2x = 6 \\ \div 2 \quad \div 2 \\ \hline x = 3 \end{array}$$

$$\begin{array}{l} 3(3) + 4 = 17 \\ 4(3) + 3 = 15 \end{array}$$

11. Number the steps in the order needed to solve the question.

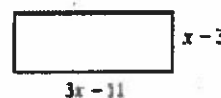
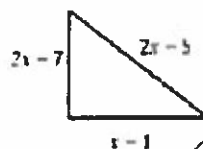
$$-\frac{2}{3}(a + 3) = \frac{5}{3}a - 19$$

2. Add $\frac{2}{3}a$ to each side.
3. Add 19 to each side.
1. Multiple a and 3 by $-\frac{2}{3}$.
4. Multiply each side by $\frac{3}{7}$.

Now, solve the equation using the above steps.

$$\begin{array}{l} -\frac{2}{3}(a + 3) = \frac{5}{3}a - 19 \\ \times \frac{3}{3} \\ \hline -2(a + 3) = 5a - 19 \\ -2a - 6 = 5a - 19 \\ +2a \quad +2a \\ \hline -6 = 7a - 19 \\ +19 \quad +19 \\ \hline 13 = 7a \\ \div 7 \quad \div 7 \\ \hline \frac{13}{7} = a \end{array}$$

12. Elin wants to fence in two different garden plots in her back yard with two rolls of fencing that are the same length. Write and solve an equation to find the value of x so that the figures below have the same perimeter.



$$\begin{array}{l} 2(2x - 7) + 2(x - 1) = 2(x - 3) + 2(3x - 11) \\ 4x - 14 + 2x - 2 = 2x - 6 + 6x - 22 \\ 6x - 16 = 8x - 28 \\ -6x \quad -6x \\ \hline -16 = 2x - 28 \\ +28 \quad +28 \\ \hline 12 = 2x \\ \div 2 \quad \div 2 \\ \hline 6 = x \end{array}$$

Test - Solving Equations

Name: key Period: _____ Date: _____

Part I: Multiple-Choice (1 point each)

B 1.) Solve: $10 + 3y = 1$

- (A) $y = -9$ (C) $y = 3$
(B) $y = -3$ (D) $y = 9$

$$\begin{array}{r} -10 + 3y = 1 \\ -10 -10 \\ \hline 3y = -9 \\ 3 3 \\ \hline y = -3 \end{array}$$

B 2.) Solve: ~~$\frac{7}{4}w = 16$~~ $\frac{7}{4}w = 16$

- (A) $w = 4$ (C) $w = 14$
(B) $w = 28$ (D) $w = 112$

A 3.) Solve: $\frac{x}{2} - 5 = -3$ ~~$\frac{x}{2} = 2$~~ $x = 4$

- (A) $x = 4$ (C) $x = -4$
(B) $x = 1$ (D) $x = -16$

D 4.) Solve: $4q - 2 = 22 - 8q$

- (A) null set (C) $q = 6$
(B) $q = -6$ (D) $q = 2$

$$\begin{array}{r} 4q - 2 = 22 - 8q \\ +8q +8q \\ \hline 12q - 2 = 22 \end{array}$$

$$\begin{array}{r} 12q - 2 = 22 \\ +2 +2 \\ \hline 12q = 24 \end{array}$$

$$\begin{array}{r} 12q = 24 \\ 12 12 \\ \hline q = 2 \end{array}$$

B 5.) Solve: $-3(p + 2) = -30$

- (A) $p = -\frac{32}{3}$ (C) $p = -8$

$$\begin{array}{r} -3p + 6 = -30 \\ +6 +6 \\ \hline -3p = -36 \\ -3 -3 \\ \hline p = 12 \end{array}$$

$p = 8$

- (B) $p = 8$ (D) $p = \frac{32}{3}$

B 6.) Solve: $4(2-k) = 2(k+7)$

(A) $k = -3$

(B) $k = -1$

$8 - 4k = 2k + 14$
 ~~$+4k$~~ ~~$+4k$~~

(C) $k = 3$

(D) $k = 11$

$8 = 6k + 14$
 ~~-14~~ ~~-14~~
 $-6 = 6k$
 $-1 = k$

C 7.) Simplify: $5 + 4(2 + 3x)$

(A) $3x + 11$

(B) $3x + 13$

(C) $12x + 13$

(D) $12x + 8$

$5 + 8 + 12x$
 $13 + 12x$

D 8.) Solve: $2(x+3) = 2x+6$

(A) $x = 6$

(B) null set

(C) $x = -3$

(D) identity

$2x + 6 = 2x + 6$

C 9.) Solve: $-3y - 4y + 9 = 9 - 7y - 12$

(A) identity

(B) $x = -2$

(C) null set

(D) $x = -\frac{12}{24}$

$-7y + 9 = -3 - 7y$
 ~~$+7y$~~ ~~$+7y$~~
 $9 = -3$

A 10.) Solve: $p + 13 = 8$

(A) $p = -5$

(B) $p = 5$

(C) $p = 21$

(D) $p = 104$

$p + 13 = 8$
 ~~-13~~ ~~-13~~
 $p = -5$

Part II – Open Response (2 points each)

11.) Solve: $5.6x + 22.1 = -2.1 + 7.6x$

$$\begin{array}{r} \cancel{5.6x} + 22.1 = \cancel{-2.1} + 7.6x \\ \hline 22.1 = 2.1 + 2x \\ \hline 24.2 = 2x \\ \hline 12.1 = x \end{array}$$

12.) Solve: $12(x+3) = 8x + 36 + 4x$

$$12x + 36 = 12x + 36$$

Infinitely
Many
Solutions

13.) Solve: $\frac{x+3}{2} = 6(2)$

$$\begin{array}{r} x+3 = 12 \\ \hline x = 9 \end{array}$$

Part III – Extended Response (3 points each)

14.) Solve: $2(3x+1)+2x=8x+4$

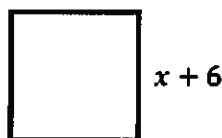
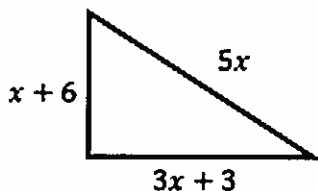
$$6x+2+2x=8x+4$$

$$\cancel{8x}+2=\cancel{8x}+4$$

$$2=4$$

No Solution

- 15.) Write and solve an equation to find the value of x that makes the perimeter of the figures below equal.



$$(x+6) + (5x) + (3x+3) = 4(x+6)$$

$$9x+9=4x+24$$

$$\begin{array}{r} 9x+9=4x+24 \\ -4x \quad -4x \\ \hline 5x+9=24 \\ -9 \quad -9 \\ \hline 5x=15 \end{array}$$

$$5x=15$$

$$\begin{array}{r} 5x=15 \\ \hline x=3 \end{array}$$

16.) Solve: $8(x+2)-3(x-4)=6(x-7)+8$

$$8(x+2)-3(x-4)=6(x-7)+8$$

$$8x+16-3x+12=6x-42+8$$

$$\begin{array}{r} 5x+28=6x-34 \\ -5x \quad -5x \\ \hline 28=-x-34 \end{array}$$

$$\begin{array}{r} 28=-x-34 \\ +34 \quad +34 \\ \hline 62=-x \end{array}$$

$$62=x$$

***Bonus* (1 point)**

Solve: $\frac{3}{4}\left(8x + \frac{16}{3}\right) = -2(x+13)$

$$\begin{array}{r} 6x + 4 = -2x - 26 \\ +2x \quad \quad +2x \end{array}$$

$$\begin{array}{r} 8x + 4 = -26 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} 8x = -30 \\ 8 \end{array}$$

$$x = \frac{-30}{8}$$

$$x = -\frac{15}{4}$$